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Time and Quantum Gravity

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The role of time in the interpretation of quantum mechanics and quantum gravity are analyzed, and changes to the form of quantum gravity to make it interpretable are suggested.

How can one interpret the "wave function of the universe" which arises in the canonical approach to quantum gravity via the "Wheeler-DeWitt" equation? In particular, is the absence of time in that approach a key to the difficulties which have surrounded that approach? Can one recast quantum gravity in a form which includes an explicit time? These concerns have led R. Wald and myself to examine the role of time in the interpretation of ordinary quantum mechanics. Out of this reexamination has arisen the feeling that one needs to reintroduce a coordinate time explicitly into the wave function of quantum gravity, rather than, as is usually done, to write the wave function as explicitly independent of the coordinates. One implication of this approach is that the wave function will not obey the usual constraint equations exactly. It turns out that there exists at least one model which is presented here in which the constraints are almost satisfied, with the exception of an unknown cosmological constant[2].

In ordinary quantum mechanics, time plays a number of roles. On one level, in the Schrödinger equation, time is that which drives the dynamics of the system, which creates and destroys correlations between the various dynamical degrees of freedom of the physical system under consideration. However, this is not the aspect I want to emphasize here as it also plays a crucial role in the interpretation of quantum mechanics.

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[2]See Unruh and Wald (1989) and Unruh (1989), where the ideas presented here are elaborated. An earlier version of the present paper is Unruh (1988).

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There are, in particular, two roles which time plays in ordinary quantum theory which I want to emphasize here. These two are that time gives us an ordering parameter on the experiments that have been done, and it divides the sets of possible observations into sets which are internally mutually contradictory. To clarify the ordering property of time, it is best to give an example.

A professor has his favorite spin-1/2 system which he stores in his lab, being careful to ensure that it is completely shielded from any outside influences such as a magnetic field (i.e., the Hamiltonian for the spin is zero). One day he goes in at 9 o'clock and measures the spin of this particle in the z direction and finds the value to be $+1/2$. Later that day, at 11 o'clock, he goes into the lab and measures the y value of the spin and finds it to be $+1/2$. Now his graduate student comes to him, and says that she measured the spin at 10 o'clock in a direction which lay in the y–z plane, and which made an angle of θ with the y direction. She asks her professor if he could tell her what the probability is that the value she measured was $+1/2$. Had she asked before he had made his measurement at 11, we could all have answered her easily. However, the knowledge of the outcome of the experiment at 11 has surely changed the probabilities. For example, if θ had been 0 (i.e., $S_\theta = S_y$), then the probability must be unity that she would have measured S_θ to be $+1/2$, since any other value would not have led to the value of $+1/2$ for the S_y measurement which was made at 11. Similarly, had she chosen $\theta = \pi/2$ ($S_\theta = S_z$), the probability must again have been unity to get 1/2 because of the prior $S_z = 1/2$ result at 9 o'clock.

In particular, it is possible to calculate the probability for an arbitrary angle θ and one finds

$$P(S_\theta = 1/2) = (1 + \cos\theta + \sin\theta + \sin 2\theta)/(2 + \sin 2\theta)$$

It is of interest to note that there exists *no* density matrix ρ such that

$$P(S_\theta = 1/2) = \mathrm{tr}(\rho(S_\theta + 1/2))$$

where $S_\theta + 1/2$ is the projection operator onto the state with $S_\theta = 1/2$. There thus exists no wave function which encodes the information which we already have about the system (namely the outcomes of the 9 and 11 o'clock experiments), and which allows us to calculate the probabilities of the outcomes of the experiment that was done on the system between those two times. Note also that this would not have been true if the student had done her experiment at 12 o'clock, say. Then we know how to calculate the probabilities

$$P(S_\theta = 1/2) = \cos^2 2\theta$$

and the wave function which is the $+1/2$ eigenstate of S_y will give this result.

In this particular problem, time has had no dynamical role ($H = 0$), but it plays a crucial role in the interpretation of the experiments.[3] It has provided an ordering parameter on the experiments, an ordering parameter which was crucial to our being able to answer the question raised by the student. Had she not told us when she did the experiment, we would not have been able to answer her question at all. It is, furthermore, the absence of such an ordering parameter which is one of the key problems in interpreting the usual "wave function of the universe" arising out of the usual approach in canonical quantum gravity.

The second role that time plays is encoded in the aphorism "Time is that which allows contradictory things to occur," or, in the words of a recent Family Circus comic strip, "Time is God's way of preventing everything from happening at once." Because of the concern expressed by Herakleitus on the role of time and on the constant clash between, and the expression of, opposites in time, I will call this the Herakleitian property of time. At any *one* time, the statement that a cup is both green and red makes no sense; these are mutually contradictory attributes. At any one time, a single particle can have only one position. However, at different times one particle can have many different positions, as can the cup have many different colors. This feature of time is again of crucial importance in the structure of quantum theory. It is the uniqueness of position for a particle, the mutually contradictory nature of many positions, that gives us the Hilbert space structure for the wave function. It is because positions x and y are mutually contradictory outcomes for the position at any one time that we can say that the probability of obtaining either x or y is the sum of the probabilities of x and y. If we have a complete set of such mutually contradictory outcomes (i.e., the outcome of the measurement *must* be one of the outcomes in our set—e.g., the particle must have some position), then the probability that some one of the outcomes is obtained must be unity. Thus, the sum of probabilities over all possible outcomes is 1. Having postulated that the probability of some one outcome is $|\Psi(x)|^2$, this gives us the requirement that $\int |\Psi(x)|^2\, dx = 1$. If one experiment could have given us two separate values for the position, we could no longer assume that the probability of x or y is the sum of their separate probabilities. It is time in its Herakleitian aspect that allows us to regard the measurements as mutually exclusive. At different times, the particle can have an arbitrary number of

[3]For a formula which gives the probabilities of some outcome, given the knowledge of the outcome of other experiments, both before and after the one of interest, see Unruh (1986). In that same volume is a very unusual paper by Aharanov *et al.* (1986) in which they show that such intermediate-time predictions can produce some very unintuitive results, namely the possibility of measuring a component of the spin of a high-spin system which is larger than the maximum possible value.

positions. If we do not specify at how many times we are making the measurement of position, we will have no structure that we can put on the space of wave functions. It is the exclusivity at one time that physically gives us the mathematical Hilbert space.

It is these two roles of time that are missing in the usual formulation of quantum gravity.[4] The wave function obeying the Wheeler-DeWitt equation is explicitly independent of time. What, then, is the object in the theory which is to play these roles of time?

There are at least two approaches to this question. The first says that one of the degrees of freedom of the problem is to be regarded not as a dynamical variable at all, but rather is to play the role of time. One is to choose one of the variables, such as the trace of the extrinsic curvature or the scale factor g, as being "time." In the quantization process, this variable is not to be quantized, but left a C-number.

The alternative approach, recently advocated by Hartle, is to regard all variables on an equal footing, choosing not a special time, but rather replacing the role of time by the readings of a clock (the "clock" being one of the dynamic systems one is examining.)

Both approaches lead to problems. The key difficulty with the second choice is that we know that even for ordinary quantum systems, no ideal clocks exist. Because of the positivity of the Hamiltonian for all real dynamical systems, the probability of a clock's stopping (i.e., showing the same reading for two different times) or even of running backward is always nonzero [see Unruh and Wald (1989) and Unruh (1988, 1989) for further details]. One therefore has on a fundamental level a problem with both of the properties that one needs time to have in order to be able to interpret the wave function.

The key problem associated with the first is that no good variable to use as time has ever emerged from the consideration of quantum gravity. Most suffer from the problem that even classically they do not act much like time. The volume of the universe, proportional in the usual Robertson-Walker cosmologies to $g^{1/2}$, grows and decreases as time goes on even in the classical solutions. The monotonicity of K, the trace of the extrinsic curvature, depends on the equations of state for the matter, and are deter-

[4]The recent concerns about the interpretation of such solutions to the Wheeler-DeWitt equations arose out of the proposal by Hawking and Hartle (1983) for a possible "wave function for the universe." J. Hartle has recently published a number of papers and preprints in which he examines the role of time in ordinary quantum mechanics and in quantum gravity. Note in particular the lectures by him in Hartle (1986, 1987). He has also written a series of papers on quantum kinematics of spacetime (Hartle, 1988a–c) which are a lucid treatment of the subject of time and have had a very strong influence on my thoughts on the subject, even though our approaches are to an extent in conflict. I would like to thank him very much for having let me see drafts of these before publication.

mined by global considerations. Furthermore, we are not free to choose the variable which we will regard as time arbitrarily. Different choices can lead to different quantum mechanics.[5]

To illustrate, let me examine one of the standard toy models that is often used in these discussions of quantum gravity. One of the key features of gravity is its invariance under coordinate transformations. I will transform a standard dynamical system with dynamical variable x into a time-reparametrization-invariant system. In particular, I will do this by introducing a new dynamical variable T and an arbitrary time coordinate τ. Let us take the original system to be described by a Lagrangian $L(dx/dt, x)$. We can make it time-reparametrization-invariant by introducing the new variable T and time τ and defining a new dynamical Lagrangian by

$$L'(\dot{T}, T, \dot{x}, x) = \dot{T}L(\dot{x}/\dot{T}, x) \tag{1}$$

where the dot denotes $d/d\tau$. The classical dynamics of the x system will be identical to that of the original, with x now being a function of $T(\tau)$ rather than t.

We can find the Hamiltonian associated with L', and we get

$$H' = \dot{T}[P_T + H(P_x, x)] \tag{2}$$

where $H(P_x, x)$ is the Hamiltonian associated with L. Usually we would be able to eliminate $\dot{T}$ with respect to P_T, but the defining equation for P_T from the Lagrangian is just

$$P_T = -H(P_x, x)$$

Now, $\dot{T}$ can be regarded as an arbitrary function of τ and so it is usually written as $N(\tau)$. We thus get the final form of the Hamiltonian

$$H' = N[P_T + H(P_x, x)]$$

Hamilton's equations now give us

$$\dot{T} = N, \qquad \dot{P}_T = 0$$

$$\dot{x} = N\,\partial H/\partial P_x, \qquad \dot{P}_x = -N\,\partial H/\partial x$$

In addition, if we vary the action with respect to N, we get the equation

$$P_T + H(P_x, x) = 0$$

a constraint equation on the initial data.

[5]See the very instructive paper by Rowher (1986), where he points out the problem in the usual "gauge" (coordinate) fixing procedure used in quantum gravity. In particular, the coordinate transformations between various forms of the coordinate-fixed quantum mechanics must be operator valued.

In quantizing this system, one could reverse the procedure, solve the classical equations for T and P_T, substitute back into the action, and then quantize the resulting system in which only x and P_x are the quantum dynamical variables.

Alternately, one can follow Dirac, and impose the constraint equation as an operator equation on the wave function:

$$i\,\partial\Psi/\partial T + H(P_x, x)\Psi = 0 \tag{3}$$

This has the added benefit that the resulting Ψ will also be independent of the arbitrary parameter τ and of the function $N(\tau)$. The procedure is manifestly reparametrization invariant.

There is, however, a third approach. Do not impose the constraints on the quantum level. Instead, write the usual quantum Schrödinger equation

$$i\,\partial\Psi/\partial\tau = N[P_T + H(P_x, x)]\Psi \tag{4}$$

The wave function will depend on T and x *and* on the combination $N\,d\tau$. This latter combination is also reparametrization invariant, because under reparametrization N and τ both change.

Our wave function now contains some unknown functions, N and τ. How can we use it to make predictions, since by assumption τ is arbitrary and N is unknown? The answer is that when we make a measurement, we need both to determine the time T and measure the variable x. Assuming that we set up the initial state of the system properly (i.e., an eigenstate of T at the initial τ which we can take as 0), our subsequent measurement of T will tell us what the combination $N\,d\tau$ is. We have

$$\Psi(N\,d\tau, T, x) = \delta(T - N\,d\tau)\phi(N\,d\tau, x)$$

and thus our measurement of T gives us $N\,d\tau$, and the state of the x system, given our measured value for T, can be written as $\phi(T, x)$. This is true for the fixed value we measured for T, but the amplitude ϕ for the x system is just that which one would have calculated if one had used equation (3), the constraint equation.

Because the parameter time τ gives one the usual ordering and Herakleitian properties one needs, there is no trouble in interpreting Ψ or ϕ. The integral over the former function squared with respect to T and x, or over the latter with respect to x, gives one the usual normalization.

Let us contrast this with the second approach to quantization, where one imposes the constraint as a quantum condition on the wave function [equation (3)]. Here one would want to interpret $\Psi(x, T)$ in the usual way, as giving, for example, the amplitude of finding the particle with position x at time T. The integral $|\Psi(x, T)|^2\,dx = 1$ would be the usual normalization. The question one has to ask is what right we have to do this. T is not the time, in the Herakleitian sense. It but is simply one of the dynamic degrees of freedom of the system. Why are we claiming that at a given T, the various possibilities for x are mutually exclusive?

This is highlighted if we make a canonical transformation on the x, T variables, defining, for example,

$$Y=(T-x), \qquad P_Y=P_T, \qquad X=x, \qquad P_X=P_x+P_T$$

We again get a "constraint" equation for $\tilde{\Psi}$,

$$[P_Y+H(P_X-P_Y, x)]\tilde{\Psi}=0$$

Now, however, we cannot interpret $|\tilde{\Psi}(X, Y)|^2=|\Psi(x, t-x)|^2$ as the probability of finding the position X for the particle at "time" Y. The various values of x are no longer mutually exclusive at a given value of Y. The x can have many different values at the same Y, or it could even have no value at a given Y. Hartle (1986, 1987, 1988a–c) has in fact argued that the average number different values that x will have for a given Y is infinite. One has thus lost the whole reason for the Hilbert space structure in terms of probabilities of outcomes of experiments.

In this simple example, it is of course clear that there is a "right" way to do the quantization. T is the preferred coordinate, for example, because the momentum conjugate to T occurs linearly in the Hamiltonian. The problem occurs in full force in quantum gravity, however, because there exists no preferred dynamical degree of freedom which one could single out as the time.

It is often stated that the constraint is the direct consequence of reparametrization invariance. In the Hamiltonian of equation (2) we have the additional parameter N (which was originally $\dot{T}$). One argues that under a change of parametrization of the theory, $\tau'=f(\tau)$, one would expect the theory to remain the same. Under such a reparametrization, N changes, leading to the idea that the theory should be invariant under arbitrary changes in N. It is the variation with respect to N in the action which leads to the "constraint" equation

$$P_T+H(P_x, x)=0.$$

Thus this equation is interpreted as being the direct consequence of reparametrization invariance of the theory. In the quantum system, imposition of the constraint is seen as imposing the demand of reparametrization invariance on the quantum theory.

There is, however, a problem with this idea. If we go back to the Lagrangian approach and institute a reparametrization of τ such that the initial and final values of τ are left alone, one finds that the requirement of reparametrization does not lead to any constraint. Instead it leads to the condition

$$dH(x, p)/d\tau = 0$$

One obtains this whether or not one regards T as a dynamic variable. Support for this view that conservation of $H(P_x, x)$ is the true consequence of reparametrization invariance also arises when one notices that the classical dynamics is left unchanged by adding a constant to the original Lagrangian. This leads to the new "constraint" equation

$$P_T - H(P_T, x) = \text{const}, \quad \text{not } 0$$

Thus, reparametrization invariance does not imply the constraint.

This observation has an immediate analog in the gravitational case. There, reparametrization invariance (coordinate invariance) implies not the equations $G^0_\mu = 0$, the usual constraints of general relativity, but $G^{\mu\nu}_{;\nu} = 0$, the Bianchi identities.

Let us return to our toy problem. The variable T is somewhat strange. It was introduced as a dynamical variable, but it is not really the variable of any physical system. If we had 1000 different "x" systems, they would all share the same T. It is not clear exactly how one would couple to or measure this dynamic variable. The question thus arises, is there any other way of introducing reparametrization invariance into our toy problem? We would like to have a toy problem with such invariance because we eventually want to understand quantum general relativity, where coordinate changes are the rule. The answer is yes. Instead of regarding T as a dynamic variable, we simply introduce a C-number function N into the Lagrangian

$$\hat{L}(\dot{x}, x) = NL(\dot{x}/N, x)$$

Under a change of parametrization $\tau' = f(\tau)$, we demand that $N' = N/(df/d\tau)$. The Hamiltonian becomes

$$H'(P_x, x) = NH(P_x, x)$$

In this case the classical equations of motion do not imply any constraint equations, and the variation of the action to give $H = 0$ is completely inappropriate.

Quantization is straightforward in that we get the Schrödinger equation

$$i\, d\Psi/d\tau = NH(P_x, x)\Psi$$

Again, Ψ will be a function of the τ, but only in the combination $N\, d\tau$. This combination is obviously reparametrization invariant.

Again one asks how one can make predictions with this wave function. The wave function contains both an arbitrary parameter τ and a function N which is unknown and unmeasurable.

At this point let us remark that this is a kind of problem which is well known in classical general relativity. There again the results of calculations give one objects which depend on arbitrary parameters, the spacetime coordinates. It was historically one of the hardest battles for the field to recognize that those solutions in terms of coordinates were in themselves useless. One had to recast those predictions into forms in which the coordinates could be eliminated from the problem. One used some of the dynamic variables to specify where one was physically in the spacetime, and then made predictions about the other variables.

A similar procedure works here. Let me give an example. Let us introduce another system, which I will call a clock, with dynamic variables D and P_D. The system is designed so that the dynamics of the variable D make it depend, say, linearly on the unknown quantity $N\,d\tau$, at least over some time period, and to some degree of approximation. Again, at some time τ, which is unspecified *a priori*, one makes a measurement of D. Since the D system is a genuine dynamical system, one can imagine how one would measure D (in contrast with the case for the T variable in the previous model). One can then use the value obtained from that measurement to determine what $N\,d\tau$ is. This allows one then to make predictions about the other variables in the system.

This procedure is well defined classically. One can design realistic physical apparatus one of whose degrees of freedom is an exact linear function of the time. However, in the quantum case, additional difficulties arise. The ability to use the measured value of D to determine $N\,d\tau$ will depend sensitively on the state of the clock. The state must be such that the determination of $N\,d\tau$ from D is statistically significant.

Let us take an example. Let the Hamiltonian for the system be

$$H(P_D, D, P_x, x) = P_D^2/2M + h(P_x, x)$$

Choose the initial state of the system to be

$$\Psi_0(D, x) = \exp[-(D^2/\delta_0^2 + iMvD)]\,\phi(x)$$

Defining $t = N\,d\tau$, we find that

$$\Psi(t, D, x) \propto \exp[-(D - vt)^2/\delta^2(t) + iMv(D - vt)]\,\phi(t, x)$$

where $\delta^2(t) = \delta_0^2 + 2it/M$. Now, at some unknown value for $N\,d\tau$, we measure D and find its value to be d. We can infer that $t = N\,d\tau$ is given by

$$t \approx d/v \pm \delta(d/v)/v$$

which will give us an accurate estimate for t if $\delta \ll d$. We can now replace t by d/v in the wave function for x. Note that this will be valid only if the change in the wave function for x is small over times of order δ/v. The wave function for x is then approximately given by $\phi(d/v, x)$. The amplitude for x given a measured value for D thus looks like the solution to "Wheeler–DeWitt"-type equation

$$i\,\partial\phi/\partial D = (h(P_x \cdot x)/v)\phi$$

Defining $T = D/v$, we thus would get the same equation that we got in our other toy problem.

There are significant differences between the two procedures. In this case, the relation between d and t is only approximate. The relation is obtained by statistical inference, rather than by defining the time to be D/v as an exact relation. The ability to perform this elimination of t in terms of d depends crucially on the state of the system, especially on the state of the clock. Had we chosen another initial state for the clock, e.g., an eigenstate of momentum, we would have been able to make absolutely no predictions for the x system, even after we had measured the value of the clock pointer. The predictive powers of the quantum system would have been null.

I will briefly outline a model theory for gravity which has the feature of containing an explicit external time, but still corresponds closely to ordinary gravity. Further details are available in Unruh and Wald (1989) and Unruh (1988, 1989). The theory is a revival of one suggested by Einstein (1919).

In this theory, the metric $g_{\mu\nu}$ is not an unconstrained symmetric tensor, but rather has the additional constraint that the determinant is defined to be unity. Although this could be accomplished for any solution of the usual Einstein equations by a suitable coordinate choice, the philosophy here is that this is a restriction on the variables of the theory instead. The action for gravity is now the analogue of the usual Einstein action

$$I = \int R(g_{\mu\nu})\, d^4x$$

with classical equations of motion given by

$$R_{\mu\nu} - \tfrac{1}{4}Rg_{\mu\nu} = 0$$

However, because of the Bianchi identities, one finds that any solution of these equations must also obey

$$R_{,\mu} = 0$$

or

$$G_{\mu\nu} = \Lambda g_{\mu\nu}$$

where Λ is an integration constant, rather than a given "coupling constant."

Because of the loss of the determinant of g as a potential dynamic variable, the constraints arising in the Hamiltonianization of this theory include only the "momentum constraints" and not the Hamiltonian constraint. Taking γ_{ij} as the usual spatial metric in the ADM Hamiltonian formulation of general relativity, the nonzero Hamiltonian becomes

$$H = \int H_0/\gamma^{1/2} + N^i H_i \, d^3x$$

with a nontrivial Schrödinger equation

$$i\,\partial\Psi/\partial t = \int H_0/\gamma^{1/2} \, d^3x \, \Psi$$

where H_i are the usual momentum constraint operators, and H_0 is the usual "energy" constraint function. In this formulation, the cosmological constant turns out to be the conjugate variable to the external time t.

As shown in Unruh and Wald (1989) and Unruh (1988, 1989), there also exists another secondary constraint $[H_0/\gamma^{1/2}]_{,i} = 0$ which encodes the fact that the cosmological constant is spatially independent.

Minisuperspace models of this theory have well-defined wave packets, which are normalizable and can be set up to follow semiclassical trajectories. Furthermore, because of the existence of an external time, the theory can be interpreted in exactly the way demonstrated above for normal nonrelativistic quantum theories. Because the classical equations are so close to the usual theory, this theory also has the feature that one would expect to have a well-behaved classical limit. Perhaps its key flaws are, to a general relativist like myself, the explicit breaking of coordinate invariance of the theory, and the introduction of a background, unquantized volume element to the theory. It is thus a backtracking from Einstein's vision of making geometry fully dynamic.

Let us contrast the two approaches to the wave function of quantum gravity.

(i) The usual approach defines the wave function (which I will call the WDW wave function) of the universe as an explicitly time-independent solution to the Wheeler–DeWitt equation, $H_0\Psi = 0$. Thus Ψ is a function of only the dynamical degrees of freedom of the system. Our approach defines the wave function as the solution of the usual Schrödinger equation

$$i\,\partial\Psi/\partial t = \left[\int (H_0/\gamma^{1/2} + N^i H_i) \, d^3x\right]\Psi$$

It is invariant under spatial coordinate transformations and certain general coordinate transformations, but loses general coordinate invariance under transformations which change the determinant of the metric.

(ii) Given that the outcome of a series of measurements is known, it is unclear how to encode that knowledge into the WDW wave function. No analog to the "collapse of the wave function" seems to be applicable to the WDW wave function. At best only vague references to the necessity of the Everett interpretation of quantum mechanics have been given in this context. Because of the parameter time in our proposal, the usual rules of quantum measurement theory apply.

(iii) The WDW is defined so as to satisfy the "constraint" equations. Thus, in any classical limit, one would expect Einstein's equations to be valid. In the example theory here with an explicit external time, the time coordinate leads only to an arbitrary cosmological constant, a sufficiently weak violation of Einstein's equations to be tolerated.

(iv) Because the WDW function does obey the constraints exactly, it is difficult to know what the measurable quantities in that theory are. One might expect that only those quantities which commute with the constraints, only those quantities which are invariant under coordinate transformations, are actually measurable. This would, however, be problematic, because most of the dynamic variables in terms of which the theory is defined (e.g., the metric g_{ij}) would then be unmeasurable. In fact, only those quantities which are constants of the motion would be measurable, as they have to commute with the Hamiltonian. This is at least in apparent conflict with our ability to measure time-dependant quantities.

Since the usual constraints are not satisfied in the sample theory, the Hamiltonian is not zero, and dynamic time-dependent quantities are in principle measurable. Only for those sets of measurements which would allow us to eliminate the time parameter would physical predictions for time-dependent quantities be meaningful, however. Time-independent quantities would always be measurable, and would again be physically meaningful.

(v) For the WDW function, the definition of a Hilbert space structure (an inner product related to probabilities) is probably impossible. Again, no such problem arises for our model, since the parameter time t has the ordering and Herakleitian structures needed. This is borne out by the existence of normalizable wave functions in the minisuperspace models.

Although the model presented here of a possible way of reintroducing time into the equations of canonical quantum gravity is certainly not supposed to be the final word, it does show that such a program of demanding an external time to play the ordering and Herakleitian roles is a possible route of attack on the extremely difficult problems facing the formulation of an interpretable theory of quantum gravity.

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